

Fields and Waves Crib Sheet

Basic Properties of Waves

Speed of light in vacuum $u_c \approx 3 \times 10^8 \text{ m/s}$

Phase Constant $\beta = \frac{\omega}{u}$	Wavelength $\lambda = \frac{2\pi}{\beta} = \frac{u}{f}$
Euler's Formula: $e^{j\theta} = \cos \theta + j \sin \theta$	

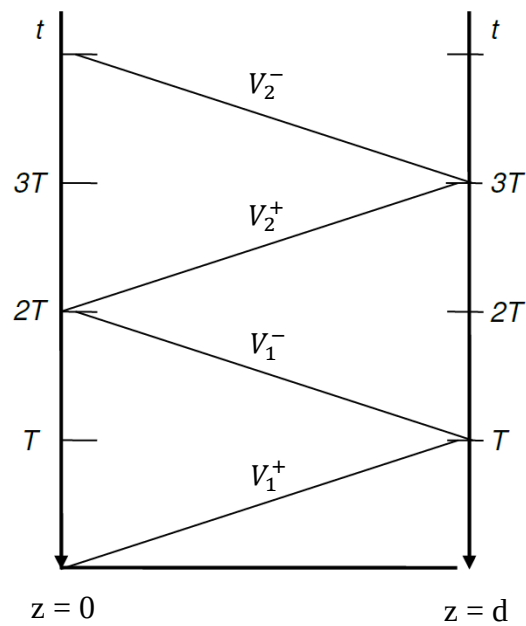
Lossless Transmission Lines

Telegrapher's Equations $\frac{\partial}{\partial z} V(z, t) = -l \frac{\partial}{\partial t} I(z, t)$ $\frac{\partial}{\partial z} I(z, t) = -c \frac{\partial}{\partial t} V(z, t)$	Voltage Wave Phasor $V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$ $V(z) = V^+ e^{-j\beta z} + \Gamma_L V^+ e^{+j\beta z}$
Characteristic Impedance $Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{l}{c}}$	Reflection Coefficient $\Gamma_L = \frac{V_0^-}{V_0^+} = -\frac{I_0^-}{I_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$
Voltage Standing Wave Ratio $\text{SWR} = \frac{ V_{\max} }{ V_{\min} } = \frac{ V_0^+ + V_0^- }{ V_0^+ - V_0^- } = \frac{1 + \Gamma }{1 - \Gamma }$	Wave Velocity $u = \frac{1}{\sqrt{lc}} = v_f \cdot u_c$
Input Impedance $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}$	Time-Average Power $P_{av} = P_{av}^i + P_{av}^r = \frac{ V_0^+ ^2}{2Z_0} (1 - \Gamma ^2)$
Z_{in} for Open Circuit Termination $Z_{in} = -jZ_0 \cot(\beta L)$	Z_{in} for Short Circuit Termination $Z_{in} = jZ_0 \tan(\beta L)$
Input Impedance: $L = \lambda/2$ $Z_{in} = Z_L$	Input Impedance: $L = \lambda/4$ $Z_{in} = \frac{Z_0^2}{Z_L}$
Incident Instantaneous Power $P^i(z, t) = \frac{ V_0^+ ^2}{Z_0} \cos^2(\omega t - \beta z + \phi^+)$	Reflected Instantaneous Power $P^r(z, t) = - \Gamma ^2 \frac{ V_0^+ ^2}{Z_0} \times \cos^2(\omega t + \beta z + \phi^+ + \theta_r)$

Lossy Transmission Lines

Voltage Wave Phasor $V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$ $V(z) = V^+ e^{-\gamma z} + \Gamma_L V^+ e^{+\gamma z}$	Propagation Constant $\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)}$ $e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$
Characteristic Impedance $Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$	Heaviside Condition $\frac{r}{l} = \frac{g}{c}$
Low-Loss Line Conditions $r \ll j\omega l; g \ll j\omega c$	Low-Loss Attenuation $\alpha = \frac{r}{2Z_0}$
Low-Loss Characteristic Impedance $Z_0 = \sqrt{\frac{l}{c}} \left(1 - j \frac{r}{2\omega l}\right)$	Attenuation on Dispersionless Lines $\alpha = r \sqrt{\frac{c}{l}}$

Bounce Diagram



Vector Math

	Symbol	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate Variables		x, y, z	r, ϕ, z	r, θ, ϕ
Vector Representation	$\vec{A}$	$A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$	$A_r \hat{r} + A_\phi \hat{\phi} + A_z \hat{z}$	$A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$
Magnitude of A	$ \vec{A} $	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$
Position Vector	$\overrightarrow{OP_0}$	$x_0 \hat{x} + y_0 \hat{y} + z_0 \hat{z}$ for $P_0 = (x_0, y_0, z_0)$	$r_0 \hat{r} + z_0 \hat{z}$ for $P_0 = (r_0, \phi_0, z_0)$	$r_0 \hat{r}$ for $P_0 = (r_0, \theta_0, \phi_0)$
Unit Vector Properties		$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$ $\hat{x} \times \hat{y} = \hat{z}$ $\hat{y} \times \hat{z} = \hat{x}$ $\hat{z} \times \hat{x} = \hat{y}$	$\hat{r} \cdot \hat{r} = \hat{\phi} \cdot \hat{\phi} = \hat{z} \cdot \hat{z} = 1$ $\hat{r} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{z} = \hat{z} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\phi} = \hat{z}$ $\hat{\phi} \times \hat{z} = \hat{r}$ $\hat{z} \times \hat{r} = \hat{\phi}$	$\hat{r} \cdot \hat{r} = \hat{\theta} \cdot \hat{\theta} = \hat{\phi} \cdot \hat{\phi} = 1$ $\hat{r} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\theta} = \hat{\phi}$ $\hat{\theta} \times \hat{\phi} = \hat{r}$ $\hat{\phi} \times \hat{r} = \hat{\theta}$
Dot Product	$\vec{A} \cdot \vec{B}$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_r B_r + A_\theta B_\theta + A_\phi B_\phi$
Cross Product	$\vec{A} \times \vec{B}$	$(A_y B_z - A_z B_y) \hat{x}$ $- (A_x B_z - A_z B_x) \hat{y}$ $+ (A_x B_y - A_y B_x) \hat{z}$	$(A_\phi B_z - A_z B_\phi) \hat{r}$ $- (A_r B_z - A_z B_r) \hat{\phi}$ $+ (A_r B_\phi - A_\phi B_r) \hat{z}$	$(A_\theta B_\phi - A_\phi B_\theta) \hat{r}$ $- (A_r B_\phi - A_\phi B_r) \hat{\theta}$ $+ (A_r B_\theta - A_\theta B_r) \hat{\phi}$
Differential Length	$d\vec{l}$	$\hat{x} dx + \hat{y} dy + \hat{z} dz$	$\hat{r} dr + \hat{\phi} r d\phi + \hat{z} dz$	$\hat{r} dr + \hat{\theta} r d\theta + \hat{\phi} r \sin\theta d\phi$
Differential Surface Area	$d\vec{S}$	$d\vec{S}_x = \hat{x} dy dz$ $d\vec{S}_y = \hat{y} dx dz$ $d\vec{S}_z = \hat{z} dx dy$	$d\vec{S}_r = \hat{r} r d\phi dz$ $d\vec{S}_\phi = \hat{\phi} dr dz$ $d\vec{S}_z = \hat{z} r dr d\phi$	$d\vec{S}_r = \hat{r} r^2 \sin\theta d\theta d\phi$ $d\vec{S}_\theta = \hat{\theta} r \sin\theta dr d\phi$ $d\vec{S}_\phi = \hat{\phi} r dr d\theta$
Differential Volume	dV	$dx dy dz$	$r dr d\phi dz$	$r^2 \sin\theta dr d\theta d\phi$

Vector Operators

	Symbol	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Gradient	$\vec{\nabla}f$	$\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$
Divergence	$\vec{\nabla} \cdot \vec{F}$	$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(F_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$
Curl	$\vec{\nabla} \times \vec{F}$	$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$	$\left(\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial(rF_\phi)}{\partial r} - \frac{\partial F_r}{\partial \phi}\right)\hat{z}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(F_\phi\sin\theta) - \frac{\partial F_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta}\right)\hat{\phi}$
Laplacian	$\vec{\nabla}^2 f$	$\frac{\partial^2 F_x}{\partial x^2} + \frac{\partial^2 F_y}{\partial y^2} + \frac{\partial^2 F_z}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 F_\phi}{\partial \phi^2} + \frac{\partial^2 F_z}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$

Electrostatics

$\oint \vec{D} \cdot d\vec{S} = \iiint \rho dV = Q_{enc}$	$\vec{\nabla} \cdot \vec{D} = \rho$
$\oint \vec{E} \cdot d\vec{l} = 0$	$\vec{\nabla} \times \vec{E} = 0$
$\vec{D} = \epsilon \vec{E}$	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

Coulomb's Law

Field of point charge q: $\vec{E} = \frac{q}{4\pi\epsilon r^2}\hat{r}$	Force on charge q_2 from charge q_1: $\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon r^2}\hat{r}$
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Potential

$\vec{E} = -\vec{\nabla}V$	$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$
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Electrostatic Boundary Conditions*

$D_{1n} - D_{2n} = \rho_s$	$E_{1t} = E_{2t}$
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*Assuming material 1 is on the top/right and material 2 is on the bottom/left.

**Capacitance and Electric Energy**

Capacitance $C = \frac{Q}{V}$	Electric Energy Density $w_e = \frac{1}{2} \epsilon \vec{E} ^2$
Total Electric Energy $W_e = \frac{1}{2} \iiint \epsilon \vec{E} ^2 dV = \frac{1}{2} CV^2$	

Laplace's & Poisson's Equations + Finite Difference

Laplace's Equation $\vec{\nabla}^2 V = 0$	Poisson's Equation $\vec{\nabla}^2 V = -\frac{\rho}{\epsilon}$
Laplace Equation Solver $V_{center} = \frac{1}{4} (\sum V_{neighbors})$	Poisson Equation Solver $V_{center} = \frac{1}{4} \left(\sum V_{neighbors} - \frac{\rho h^2}{\epsilon} \right)$

Current Flow

Microscopic Ohm's Law $\vec{J} = \sigma \vec{E}$	Resistance $R = \frac{V}{I}$
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Magnetostatics

$\oint \vec{B} \cdot d\vec{S} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
$\oint \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{S} = I_{enc}$	$\vec{\nabla} \times \vec{H} = \vec{J}$
$\vec{B} = \mu \vec{H}$	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

Magnetic Vector Potential

$\vec{B} = \vec{\nabla} \times \vec{A}$	$\psi = \oint \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$
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Flux, Flux Linkage, Inductance & Faraday's Law

Flux: $\psi = \iint \vec{B} \cdot d\vec{S}$	Flux Linkage: $\Lambda = N\psi$
$Emf = -L \frac{dI}{dt}$	$Emf = -\frac{d\Lambda}{dt}$
Differential Form of Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Integral Form of Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S}$
$L = \frac{\Lambda}{I}$	

Magnetostatic Boundary Conditions*

$B_{1n} = B_{2n}$	$\hat{n}_{21} \times (\vec{H}_{1t} - \vec{H}_{2t}) = \vec{J}_s$
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*Assuming material 1 is on the top/right and material 2 is on the bottom/left. $\hat{n}_{21}$ points from material 2 to material 1.

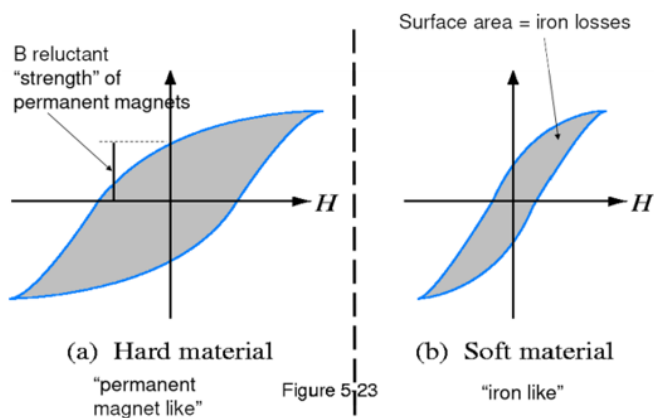
Magnetic Circuits

Magnetomotive Force (MMF): $V_m = NI$	Reluctance: $\mathcal{R} = \frac{l}{\mu A}$
$NI = \psi \mathcal{R}$	

Magnetic Energy and Force

Magnetic Energy Density: $w_m = \frac{1}{2} \frac{ \vec{B} ^2}{\mu} = \frac{1}{2} \mu \vec{H} ^2$	Total Magnetic Energy: $W_m = \iiint w_m dV = \frac{1}{2} LI^2$
Force due to Magnetic Energy Pressure: $F = -\vec{\nabla} W_m = w_m \cdot \text{Area}$	Force on a Point Charge: $\vec{F} = q(\vec{v} \times \vec{B})$

Magnetization Curves



Constants

$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
$\eta_0 = \sqrt{\mu_0/\epsilon_0} = 377\Omega$	$c \approx 3 \times 10^8 \text{ m/s}$

Full Version of Maxwell's Equations

Integral Form	Differential Form
$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$	$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint \vec{B} \cdot d\vec{S} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\oint \vec{D} \cdot d\vec{S} = \oint \rho dV = Q_{enc}$	$\vec{\nabla} \cdot \vec{D} = \rho$
Conduction Current $I_c = \iint J_c \cdot d\vec{S} = \iint \sigma(\vec{E} \cdot d\vec{S})$	Displacement Current $I_d = \iint J_d \cdot d\vec{S} = \iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$

Complex Permittivity and EM Waves in Lossy Media

Complex Permittivity $\epsilon_c = \epsilon' - j\epsilon''$, where $\epsilon' = \epsilon$ and $\epsilon'' = \frac{\sigma}{\omega}$	
Skin Depth $\delta_s = \frac{1}{\alpha}$	Intrinsic Impedance $\eta = \frac{ \tilde{E} }{ \tilde{H} }$
$u = \frac{1}{\sqrt{\mu\epsilon'}}$	$\lambda = \frac{2\pi}{\beta}$

Low-Loss Dielectrics

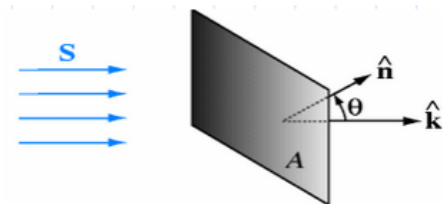
$\frac{\epsilon''}{\epsilon'} < 0.01$	$\eta = \sqrt{\frac{\mu}{\epsilon}}$
$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}}$	$\beta = \omega \sqrt{\mu\epsilon}$

Good Conductor

$\frac{\epsilon''}{\epsilon'} > 100$	$\eta = (1 + j) \frac{\alpha}{\sigma}$
$\alpha = \sqrt{\pi f \mu \sigma}$	$\beta = \sqrt{\pi f \mu \sigma}$

EM Waves and Energy

Magnetic Field $\tilde{H} = \hat{k} \times \frac{\tilde{E}}{\eta}$	Electric Field $\tilde{E} = -(\hat{k} \times \eta \tilde{H})$
Poynting Vector $\vec{S} = \vec{E} \times \vec{H}$	$P = \int_A (\vec{S} \cdot \hat{n}) dA$
$P = \vec{S} A \cos \theta$	$S_{av} = \frac{ \tilde{E} ^2}{2\eta}$
Time-Average Power Density in Lossy Media $S_{av} = \frac{ \tilde{E} ^2}{2\eta} e^{-2\alpha z}$	



Wave Polarization

Polarization Phasor Expression: $\tilde{E}(z) = (a_x \hat{x} + a_y e^{j\delta} \hat{y}) e^{-jkz}$	Linear Polarization Inclination Angle (ψ): $\psi = \tan^{-1} \left(\frac{a_y}{a_x} \right)$
For linear polarization: $\delta = 0, \pm\pi$ For circular polarization (for propagation in +z): When $\delta = +\frac{\pi}{2}$: left-handed When $\delta = -\frac{\pi}{2}$: right-handed	Elliptical Polarization Auxiliary Angle (ψ_0): $\psi_0 = \tan^{-1} \left(\frac{a_y}{a_x} \right)$
Elliptical Polarization Rotation Angle (γ): $\tan 2\gamma = (\tan 2\psi_0) \cos \delta$ (for $-\frac{\pi}{2} < \gamma < \frac{\pi}{2}$)	Elliptical Polarization Ellipticity Angle (χ): $\sin 2\chi = (\sin 2\psi_0) \sin \delta$ (for $-\frac{\pi}{4} < \chi < \frac{\pi}{4}$)

Wave Propagation, Reflection and Transmission

In Lossless Media: $\gamma = k = \frac{2\pi}{\lambda} = \frac{\omega}{u}$ Refractive Index: $n = \sqrt{\epsilon_r}$	
Wave Phasor with E pointed in x and propagating in z: $\tilde{E}_1(z) = E_0^i \hat{x}(e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z})$	Propagation Constant $\gamma_1 = k_1 = \alpha_1 + j\beta_1$
Normal Incidence Reflection Coefficient $\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_{r1}} - \sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}$	Normal Incidence Transmission Coefficient $\tau = 1 + \Gamma = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$
Snell's Law $k_1 \sin \theta_1 = k_2 \sin \theta_2$ $n_1 \sin \theta_1 = n_2 \sin \theta_2$ $\eta = \frac{\eta_0}{n}$	Parallel Polarization Reflection Coefficient: $\Gamma_{ } = \frac{\eta_2 \cos \theta_2 - \eta_1 \cos \theta_1}{\eta_2 \cos \theta_2 + \eta_1 \cos \theta_1}$
Parallel Polarization Transmission Coefficient: $\tau_{ } = \frac{2\eta_2 \cos \theta_1}{\eta_2 \cos \theta_2 + \eta_1 \cos \theta_1}$ $1 + \Gamma_{ } = \tau_{ } \frac{\cos \theta_2}{\cos \theta_1}$	Perpendicular Polarization Reflection Coefficient: $\Gamma_{\perp} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$
Perpendicular Polarization Transmission Coefficient: $\tau_{\perp} = \frac{2\eta_2 \cos \theta_1}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$ $1 + \Gamma_{\perp} = \tau_{\perp}$	Critical Angle: $\theta_c = \sin^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sin^{-1} \left(\frac{n_2}{n_1} \right)$
Perpendicular Brewster Angle: $\sin \theta_{B\perp} = \sqrt{\frac{1 - \left(\frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1} \right)}{1 - \left(\frac{\mu_1}{\mu_2} \right)^2}}$	Parallel Brewster Angle: $\sin \theta_{B } = \sqrt{\frac{1 - \left(\frac{\epsilon_1 \mu_2}{\epsilon_2 \mu_1} \right)}{1 - \left(\frac{\epsilon_1}{\epsilon_2} \right)^2}}$ if $\mu_1 = \mu_2$: $\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

* $\beta = k$ for wave propagation in free space or lossless materials